First of all, understand that roots are a subset of exponents i.e. any number with any root can be converted to an exponent. Every rule of exponent that we have learnt till now will then be applicable.

Rule 1: $\sqrt[n]{a} = a^{\frac{1}{n}}$

Example 1: $\sqrt[3]{8} = 8^{\frac{1}{3}}$

Let's see this rule in action.

Question 1: What is $\sqrt[3]{9} \times \sqrt[3]{3}$?

Solution:

$\sqrt[3]{9} = 9^{\frac{1}{3}} = (3^2)^{\frac{1}{3}} = 3^{\frac{2}{3}}$ (Using Exponents Rules we learned in the previous posts)

$\sqrt[3]{3} = 3^{\frac{1}{3}}$

$\sqrt[3]{9} \times \sqrt[3]{3} = 3^{\frac{2}{3}} \times\ 3^{\frac{1}{3}} = 3^{\frac{2}{3}+\frac{1}{3}} = 3^1 = 3$

The rules for exponents are the same whether we have an integer as the exponent or a fraction. Therefore, if you think that roots are a nightmare but exponents are fine, just convert the roots to exponents form and the question becomes utterly do-able. But, when you take that route, it takes far too much extra time. So, it makes sense to review the rules of roots (which are very similar to the rules of exponents)

Question 2: What is $\sqrt{4}$?

We know this is 2.

How?

$\sqrt{4} =\ 4^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2$

Or we can directly say that there are two 2s under the square root sign. When we remove the square root sign, we are left with a single 2.

$\sqrt{4} = \sqrt{2 \times 2} = 2$

Question 3: On the same lines, what is $\sqrt[3]{27}$?

$\sqrt[3]{27} =\ \sqrt[3]{3 \times 3 \times 3} = 3$

Since we need the third root here, the three 3s give us a single 3.

Question 4: What is $\sqrt[4]{64}$?

$\sqrt[4]{64} = \sqrt[4]{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2\sqrt[4]{4}$

Here, we need the fourth root of 64. 64 consists of six 2s. Since this is the fourth root, out of four 2s, we take one 2 out. The rest of the two 2s remain put.

Question 5: What is $\sqrt[3]{432}$?

$\sqrt[3]{432} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3} = 2 \times 3 \times \sqrt[3]{2} = 6\sqrt[3]{2}$

432 has four 2s and three 3s. Since we need the third root, three 2s are used to get one 2 out, three 3s are used to get one 3 out and a 2 is leftover under the third root.

I hope this method is clear. Let's go on now.

Rule 2: $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$

Example 1: $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

Example 2: $\sqrt[3]{7} \times \sqrt[3]{49} = \sqrt[3]{7 \times 7 \times 7} = 7$

Rule 3: $\sqrt[n]{a} \,/\, \sqrt[n]{b} = \sqrt[n]{a/b}$

Example 1: $\sqrt{8}/\sqrt{2} = \sqrt{4} = 2$

Example 2: $\sqrt[3]{49} \times \sqrt[3]{7} = \sqrt[3]{7}$

Rule 4: $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Example 1: $\sqrt[4]{\sqrt[2]{256}} = \sqrt[8]{256} = \sqrt[8]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2$

Let's look at a complicated question now.

Question 6: Given that $\frac{\sqrt[8]{512} \times (\sqrt{343})^3}{\sqrt{4} \times \sqrt[3]{\sqrt[2]{128}}} = 2^a \times 7^b$, what is the value of a and b?

(A) a = 1, b = 1/3
(B) a = 12/17, b = 2/3
(C) a = -10/21, b = 11/2
(D) a = 11/24, b = 4/9
(E) a = -25/24, b = 9/2

Let's focus on the left hand side of the equation first.

$$\frac{\sqrt[8]{512} \times (\sqrt{343})^3}{\sqrt{4} \times \sqrt[3]{\sqrt[2]{128}}}$$

Using Rule 4: $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$, we get

$$\frac{\sqrt[8]{512}\times\left(\sqrt{343}\right)^3}{\sqrt{4}\times\sqrt[3]{\sqrt[2]{128}}} = \frac{\sqrt[8]{512}\times\left(\sqrt{343}\right)^3}{\sqrt{4}\times\sqrt[6]{128}}$$

We have some big numbers here. Let's bring them down to prime factors.

512 = 2^9

343 = 7^3

128 = 2^7

$$\frac{\sqrt[8]{512}\times\left(\sqrt{343}\right)^3}{\sqrt{4}\times\sqrt[3]{\sqrt[2]{128}}} = \frac{\sqrt[8]{2\times2\times2\times2\times2\times2\times2\times2\times2}\times\left(\sqrt{7\times7\times7}\right)^3}{\sqrt{4}\times\sqrt[6]{2\times2\times2\times2\times2\times2\times2}}$$

$$\frac{\sqrt[8]{512}\times\left(\sqrt{343}\right)^3}{\sqrt{4}\times\sqrt[3]{\sqrt[2]{128}}} = \frac{2\times\sqrt[8]{2}\times\left(7\sqrt{7}\right)^3}{2\times2\times\sqrt[6]{2}}$$

The 2 in the numerator gets canceled with the 2 in the denominator.

$$\frac{\sqrt[8]{512}\times\left(\sqrt{343}\right)^3}{\sqrt{4}\times\sqrt[3]{\sqrt[2]{128}}} = \frac{\sqrt[8]{2}\times\left(7\sqrt{7}\right)^3}{2\times\sqrt[6]{2}}$$

Using Rule 1, let's convert the rest of the roots to exponents.

$$\frac{\sqrt[8]{512}\times\left(\sqrt{343}\right)^3}{\sqrt{4}\times\sqrt[3]{\sqrt[2]{128}}} = \frac{2^{\frac{1}{8}}\times7^3\times7^{\frac{3}{2}}}{2\times2^{\frac{1}{6}}}$$

Now, using the rules of exponents, we further simplify this to get

$$\frac{\sqrt[8]{512}\times\left(\sqrt{343}\right)^3}{\sqrt{4}\times\sqrt[3]{\sqrt[2]{128}}} = \frac{2^{\frac{1}{8}}\times7^{3+\frac{3}{2}}}{2^{1+\frac{1}{6}}} = 2^{\frac{1}{8}-\frac{7}{6}}\times7^{3+\frac{3}{2}}$$

$$\frac{\sqrt[8]{512}\times\left(\sqrt{343}\right)^3}{\sqrt{4}\times\sqrt[3]{\sqrt[2]{128}}} = 2^{-\frac{25}{24}}\times7^{\frac{9}{2}}$$

Therefore, a = -25/24 and b = 9/2

Answer (E)

Even though it looks really complicated, I would suggest you to go step by step. Just use the rules we have learned and you will get to the answer.